



The Research Base for Straight Curve™ Mathematics

Technical Paper #18

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Introduction

Technology-Infused Tools for Teachers

Straight Curve™ Mathematics was developed to fill a specific need: to provide classroom teachers with resources that can help students develop a deeper conceptual understanding of mathematics. *Straight Curve Mathematics* leverages the best of what technology has to offer; namely, technology allows the easy manipulation of models difficult to reproduce offline, it provides convenient facilitation of presentations and assessments, and it is a natural draw for young students. However, technology is not an answer on its own. Technology is only useful in the hands of a skilled teacher, and in any case the reality of classrooms today is that technology, no matter how compelling, is not always an option. To that end, every lesson in *Straight Curve Mathematics* is infused with both rich teacher support materials and with recommendations for offline alternatives. In short, *Straight Curve Mathematics* is designed to be a flexible set of instructional tools that guide teachers to upgrade their skills.

This white paper describes the research basis for *Straight Curve Mathematics*. It lays out the specific problems the instruction is designed to address, the key principles used to address them, and the research base behind those principles.

Problem Areas in Elementary Mathematics Instruction

Three Critical Problem Areas

In 1989, the National Research Council (NRC), which is the principal operating agency of the National Academy of Sciences in providing services to the U.S. government, began publishing a series of reports underscoring the need for everyone in an information-driven society and economy to achieve mathematics fluency. Despite this call, and despite fifteen intervening years of reform advocacy by organizations such as the National Council of Teachers of Mathematics (NCTM), recent NRC reports highlight many entrenched problems facing American mathematics educators.

There is hope. Researchers continue to provide new evidence that the innovative practices recommended by NCTM and others do work. While for decades many Americans have declared, almost proudly, “I’m not good at mathematics,” the changing landscape of educational emphasis will, with concerted effort, one day make such admissions rare, and certainly not something to be proud of.

One day, America will be a nation of mathematics—not mathematics in the sense of academics who devote their lives to advanced calculus, but mathematics in the sense of people who embrace mathematics as a useful, flexible tool and not as an abstraction to be feared or avoided. To get there will require some profound changes, however, in how schools approach mathematics instruction.

Below are three key problem areas in elementary school mathematics that have profoundly influenced the creation of *Straight Curve Mathematics*.

Shallow Curricula

One area in which schools need to improve, according to the NRC (2001), is in the depth of their mathematics curricula. The NRC comments:

State, national, and international assessments conducted over the past 30 years indicate that, although U.S. students may not fare badly when asked to perform straightforward computational procedures, they tend to have a limited understanding of basic mathematical concepts. They are also notably deficient in their ability to apply mathematical skills to solve even simple problems. (p. 4)

The NRC credits this limited understanding of basic concepts to mathematics curricula it characterizes as “shallow, undemanding, and diffuse in content coverage” (p.4). Whereas many other, higher achieving countries tend to go deeper into fewer concepts, American curricula cover more concepts, forcing superficial coverage.

Widely used textbooks exacerbate the problem. The NRC (2001) notes, “To be sold nationwide, a textbook needs to include all the topics from the standards and curriculum frameworks of at least those influential states that officially adopt lists of approved materials. Consequently, the major U.S. school mathematics textbooks, which collectively constitute a de facto national curriculum, are bulky, address many different topics, and explore few topics in depth” (p. 37).

Uninformative Assessments

A second weakness of American elementary mathematics programs identified by the NRC is a lack of meaningful assessment. To be sure, American children face a great number of assessments, many of them carrying serious consequences for students, teachers, and schools. However, these summative assessments do not provide specific information to teachers that they can use to inform their day-to-day classroom teaching.

Further, current forms of assessment encourage a specific style of instruction that emphasizes procedures and repeated practice over deeper development of mathematics concepts.

Underprepared Teachers

Finally, the NRC (2001) notes, “the preparation of U.S. preschool to middle school teachers often falls far short of equipping them with the knowledge they need for helping students to develop mathematical proficiency” (p. 4). In an important study in the 1990s, Liping Ma compared the conceptual mathematical knowledge of American teachers with their Chinese counterparts. She gave a group of above average American elementary teachers problems like $1\frac{3}{4} \div \frac{1}{2}$ and asked them to come up with models or scenarios to help students understand this problem conceptually. Almost none of the American teachers could give a real-world scenario that might lead to this calculation—with many of the teachers giving scenarios that really illustrated different problems like $1\frac{3}{4} \times \frac{1}{2}$. In contrast, the vast majority of Chinese teachers in the study could give a real-world

application of this equation, with many teachers generating multiple scenarios. Ma notes that teachers who only understand mathematics as a series of rote procedures will only be able to teach it as a series of rote procedures (Ma, 1999).

Deep conceptual understanding of mathematics is important, but teachers also need professional guidance about teaching strategies. Absent quality professional development, teachers tend to teach as they have always taught. To be sure, this inertia protects classrooms from premature adoption of educational fads, but it also excludes teachers from taking advantage of techniques developed or validated by rigorous research.

The NRC, in fact, cites three types of preparedness needed by teachers: knowledge of mathematics, knowledge of students, and knowledge of instructional practice. In terms of knowledge of mathematics, the NRC notes (2001):

Many recent studies have revealed that U.S. elementary and middle school teachers possess a limited knowledge of mathematics, including the mathematics they teach. The mathematics education they received, both as K–12 students and in teacher preparation, has not provided them with appropriate or sufficient opportunities to learn mathematics. As a result of that education, teachers may know the facts and procedures that they teach but often have a relatively weak understanding of the conceptual basis for that knowledge. Many have difficulty clarifying mathematical ideas or solving problems that involve more than routine calculations. For example, virtually all teachers can multiply multi-digit numbers, but several researchers have found that many prospective and practicing elementary school teachers cannot explain the basis for multidigit multiplication using place-value concepts and the underlying properties for adding and multiplying. In another study, teachers of fourth through sixth graders scored over 90% on items testing common decimal calculations, but fewer than half could find a number between 3.1 and 3.11. (p. 372)

As for knowledge of students and instructional practices, the NRC, among others, has observed the remarkable consistency of instructional practices in mathematics over the past 100 years. Despite real advances in what we know about mathematics pedagogy, surprisingly little has changed in the classroom.

A Solution

Straight Curve Mathematics is designed specifically to address the three problem areas outlined above. It helps teachers:

- Go deeper with key mathematical concepts. Rather than try to address a broad array of objectives in a shallow way, *Straight Curve Mathematics* targets those objectives deemed hardest for students to learn and hardest for teachers to teach. The *Straight Curve Mathematics* instructional design team began with the Open Book analysis of mathematics strands (Educational Market Research, 2000) and from there engaged external subject matter experts to narrow objectives.

- Make informed instructional decisions. *Straight Curve Mathematics* includes several levels of formal assessment as well as guidance to teachers about how to informally assess their students throughout the learning process.
- Become more comfortable with mathematics and with best practices for teaching mathematics. Embedded teacher support materials provide the right depth of information for teacher development. Further, *Straight Curve Mathematics* was designed to work in conjunction with expert, experienced professional development support provided by PLATO[®] Professional Services.

Five Key, Research-Based Principles Driving *Straight Curve Mathematics*

Five Key Principles

The development team behind *Straight Curve Mathematics* was guided by five key principles derived from the best available research on elementary mathematics instruction:

- Mathematics principles are best learned and internalized by learners when they are partners in generating mathematical hypotheses and algorithms.
- Classrooms that create a mathematics community produce in learners a greater ability to tackle difficult mathematics concepts and then reapply them to real-world problems.
- Teachers' knowledge of mathematical content and how they apply that knowledge in the classroom impacts student learning.
- Mastery of mathematics concepts and principles requires extended, meaningful, and engaging practice.
- Assessment should provide teachers with meaningful information that guides their instructional decisions (and students with meaningful information that guides their performance).

These principles are covered in detail below.

Mathematics principles are best learned and internalized by learners when they are partners in generating mathematical hypotheses and algorithms.

One difference between U.S. classrooms and many international classrooms is how concepts and algorithms are introduced to students. The Third International Mathematics and Science Study (TIMSS) was designed to measure differences between mathematics and science achievement in 41 countries (Stigler & Hiebert, 1999). One illuminating component of this carefully designed study was a video-based comparison of the teaching practices of German, Japanese, and American teachers. TIMSS researchers found that German and, especially, Japanese teachers tend to center their instruction around thought-provoking problems. A typical mathematics session in these high-performing countries might look like this:

1. The teacher poses a thought-provoking problem.
2. Students wrestle with the problem, often in groups.
3. The class reconvenes in a mathematics congress, where approaches to solving the problem are discussed.
4. The teacher guides the class toward reaching one or more conclusions.
5. Students practice solving similar problems.

The researchers contrast this approach to classrooms in the United States, where teachers tend to lead by teaching a particular concept or procedure, then ask learners to solve relevant problems. The international approach, according to the TIMSS researchers, promotes deep learning of concepts, whereas the American approach promotes application of a concept or procedure to problems without deep cognitive processing.

Other research efforts support the conclusions of the TIMSS researchers. Hatfield, et al. (2000), for instance, point to the work of Sylwester (1995), Jensen (1998), and Dehaene (1997) showing how the surge of brain research done in the 1990s supports the notion that the dense interconnected web of neurons that constitutes any person's knowledge of mathematics is richest when students work to make sense of mathematics and its role in the surrounding world, meaning students learn better when they are active participants in learning rather than passive recipients. As learners struggle to make sense of concepts, they create many more connections to prior knowledge than they do with traditional instruction. The more connections that learners can make to the new knowledge, the easier that knowledge will be to retrieve in a variety of situations (Schwartz & Bransford, 1998).

While the superiority of active over passive learning has been accepted by many researchers as *prima facie* true since at least the time of Dewey, experimental research supporting this mode of learning has been lacking. Fortunately, that is changing. For example, Capon and Kuhn (2004) recently used an experimental design to show that problem-based learning holds significant advantages in the classroom. As they put it, "The best way to describe this effect is to say that

students who experienced problem-based instruction more often were able to integrate newly acquired concepts with existing knowledge structures that had been activated. In everyday language, they demonstrated understanding” (p. 74). Capon and Kuhn have found in their studies that introducing difficult concepts in the form of problems leads to deeper understanding. That’s why, as elaborated below, *Straight Curve Mathematics* lessons include investigation-based activities that ask learners to solve rich problems in small groups, then discuss their solutions with the larger mathematics community in the classroom.

John Van de Walle (2004) calls such rich understanding “relational”—that is, the new knowledge becomes related to many other pieces of knowledge. He points to six benefits of relational knowledge supported in the research literature:

- Intrinsic motivation. Learning by rote often requires external forms of motivation, while acquiring rich new knowledge that connects well with existing ideas is inherently enjoyable. As Van de Walle puts it, “The new knowledge makes sense; it fits; it feels good” (p. 26).
- Easier to remember. Cognitive science teaches us that the better connected a given piece of knowledge is to other pieces of knowledge, the easier it is to recall. Further, integrated knowledge can be generated from related concepts if needed, which is certainly not true of isolated facts.
- Lays a foundation for future learning. The more deeply basic concepts are understood, the easier it is to build on those concepts.
- Improved problem solving. Learners can more easily generalize their knowledge to novel problems.
- Self-generative. Once learners become accustomed to generating their own algorithms, they begin to generate algorithms spontaneously.
- Improved attitudes. Learners can sense when their knowledge is a fragile construct. Deep understanding leads to “a definite sense of ‘I can do this! I understand!’” (p. 26).

How this principle is embodied in *Straight Curve Mathematics*

Every lesson in *Straight Curve Mathematics* contains five different activities, each with a distinct instructional purpose:

- Mini-lesson—designed to help teachers deliver meaningful direct instruction
- Investigation—designed to challenge students to find patterns or solve a significant problem in small groups
- Workshop—whole class discussion about the Investigation
- Game—practice
- Quiz—formal practice and assessment

The principle that learners should be generating algorithms rather than just learning them is best embodied in the Investigations. Every Investigation is

divided into two parts—a short Warm-Up that either refreshes prior knowledge or teaches a small, important new concept and a Challenge that presents students with a problem to solve. While the problems vary in their contexts and forms, they all have several elements in common. Every Challenge:

- Presents a problem that can be solved in a wide variety of ways. Investigations never advise learners to use particular strategies or tools—such advice would short circuit the problem-solving process.
- Asks students to capture their problem-solving processes in their mathematics journals. In every Investigation, the final answer isn't important; the problem-solving strategies are paramount.

Students given a blank canvas to invent their own algorithms or conceptual models will, of course, come up with a variety of thoughts—some completely misguided, others brilliantly insightful, and most somewhere in between. No matter how brilliant or misguided their ideas, however, learners will feel invested in these ideas because they are their own. This investment is the key to students' development of deep understandings tied to their own worlds. At this point, teachers can help learners overcome misconceptions, and, more importantly, learners can help each other overcome misconceptions and turn their ideas into efficient algorithms. This negotiation takes place in the Mathematics Workshop, which is described below as part of the next key principle.

Classrooms that create a mathematics community produce in learners a greater ability to tackle difficult mathematics concepts and then reapply them to real-world problems.

Several researchers have written at length about the advantages of turning classrooms into mathematics communities. The transformation from mathematics class to mathematics community is a profound change. Instead of a group of students working to complete assignments, a group of students acting as a mathematics community seeks to discover and solve real-world problems. Members of a mathematics community listen to the ideas of other members, offering respectful critiques where appropriate. Members of a mathematics community see mathematics as a tool, not an obstacle.

James Hiebert and his colleagues, for instance, have written many influential works showing that group forums in which students share and discuss their ideas for how to solve a problem increases “deep understandings of mathematics” (1997, p. 43). Because students are exposed to several methods of approaching a problem other than their own, they can avoid getting into a rut—avoid getting locked into one way of thinking about something and the multiple approaches will increase their understanding of a problem. Collaboration is also closer to really “doing mathematics which “depends on communication and social interaction” and involves rigorous discussions in which assumptions, assertions, and conjectures are proposed, challenged, and defended (p. 44). A 1995 study (Qin, et al., cited in Hiebert, et al., 1997) found that “cooperative settings” proved “more beneficial for students’ learning” than “competitive or individual problem-solving activities,” which may be the case because of student exposure to a variety of methods and the opportunity to talk about why certain methods work (p. 45).

Moreover, the kinds of communication that this discussion involves can give students a chance to make their vague intuitive ideas clear—for themselves and for others (Hiebert, et al., 1997). Relatedly, the creation of “cognitive conflict” encourages students to deepen their understanding of ideas by defending them vis-à-vis the questions and conflicting ideas of others.

John Van de Walle sees such a community as one in which “students feel comfortable trying out ideas, sharing insights, challenging others, seeking advice from other students and the teacher, explaining their thinking, and taking risks” (p. 32), one in which “students evaluate their own assumptions and those of others and argue about what is mathematically true” (p. 32, citing Corwin (1996) and Lampert (1990)). Van de Walle describes the “atmosphere of respect” in two classrooms that foster such a process: fourth-grade students raise their hands and say “I would like to add to what Marcel just said” or “I disagree with Tawanna,” and second-graders use their index finger to make a “point of interest,” a polite way to disagree (p. 32, citing Smith (1996)).

The role of the teacher in this community is key to fostering this sense of respect, to making sure that correctness is determined by the logic of the mathematics (Hiebert, et. al, 1997), and to treating mistakes (in student-derived algorithms, for example) constructively, as “a natural and important part of the process of improving methods of solution” (p. 48).

By “helping children become mathematicians in a mathematics community” (Fosnot and Dolk, 2001), a mathematics community helps them become problem-solvers instead of meaningless rule-followers. This forum helps them understand that mathematicians are people who think through situations and come to conclusions, and, while the students need not reinvent the entire understanding of mathematics that took us so many centuries to develop, they can share in that active process, which fosters deeper understanding than does mere memorization. In *Straight Curve Mathematics*, this mathematics community would ensure that arrived-at algorithms do not stand as truths handed down to learners, one more set of meaningless number facts divorced from a relevant context. Because they have developed ways of solving problems through active thinking and sharing of ideas, students will be better able to apply their knowledge to the solution of future problems they encounter.

Most American classrooms are far removed from the mathematics community reality described by Fosnot, Hiebert, and others. Wood (1999), through a series of long-term studies in second grade classrooms, shows that the transformation is possible. Wood describes the process that teachers use to establish the ground rules for discourse in their classrooms. Once those norms take root, students begin to have deeper mathematical discussions. As Wood describes it,

The findings from this research indicate that creating a context for argument in classrooms requires that teachers establish expectations for children’s thinking and participation while they explain their solutions to others. Of even greater significance, however, are the expectations teachers need to establish for their students as listeners. In this study listeners were expected to follow the thinking and reasoning of others to determine whether what was presented was logical and made sense. As listeners, students were also expected to voice their disagreement and to provide reasons for disagreeing. When these classroom routines became the tacit patterns of interaction, the children no longer found it necessary to direct their cognitive attention to making sense of their social setting and could direct their mental activity to making sense of their mathematical experiences. (p. 189)

Wood calls what happens in these classrooms the “precursor to the development of mathematical argumentation” (p.189).

Maher and Martino (1996) have engaged in a series of longitudinal studies with students to find out more about how students construct mathematical ideas. They note:

We have increasingly more evidence that students, when presented with a problem task in a small-group setting, begin by constructing personal representations of the problem or, in attempting to do so, discover that they cannot. After students have attempted to build personal

representations, they frequently initiate conversation with others and compare their ideas. Several earlier studies have indicated that when students compare their ideas, they frequently modify, consolidate, or strengthen an original argument or reject an initial attempt in favor of another that appears more sensible. (p. 196)

Classrooms set up as mathematics communities build on these tendencies.

Finally, in its review of the landscape of research in mathematics pedagogy, the National Research Council (2001) points to four features of mathematics communities:

- Ideas and methods are valued and respected. Any idea brought up by any student is an opportunity for learning.
- Students feel free to explore alternative methods for solving problems.
- Mistakes are valued as opportunities for exploration and learning.
- The arbiter in all mathematical disagreements is mathematical logic.

One of the primary goals of *Straight Curve Mathematics* is to help teachers transform their classrooms into mathematics communities.

Teachers who embrace the first two principles above will create classrooms of mathematicians. However, even the idea of mathematics communities can be taken too far. The NRC (2005) cautions against possible excesses, pointing out that too much focus on student-invented algorithms, for instance, can focus too much classroom time on student-invented methods that do not generalize. Further, student-focused discussions can be meandering affairs, or they may fail to develop beyond student-to-teacher interactions. Despite these potential problems, the NRC's recommendation is clear: these problems are surmountable, and American classrooms need to move in this direction. The potential excesses or problems can be ameliorated through careful teaching. Teachers experienced in these instructional approaches quickly recognize when students are churning, and they develop strategies for moving students forward.

How this principle is embodied in *Straight Curve Mathematics*

Absent high-quality instructional materials for support, gaining the experience necessary to create mathematics communities in their classrooms is a daunting prospect for many teachers. *Straight Curve Mathematics* is specifically designed to support teachers through this transition.

First, every lesson in *Straight Curve Mathematics* devotes two entire instructional blocks to encouraging students to invent ideas they can share with a mathematics community in their classroom. In the Investigation, students are given an open-ended problem to solve in small groups, then they reconvene to discuss those solutions in the Mathematics Workshop. These instructional blocks are carefully designed to facilitate meaningful thought and discussion on a focused objective.

Further, each of these instructional blocks, like all instructional blocks in *Straight Curve Mathematics*, is augmented with extensive, easily accessed teacher support materials. These materials provide teachers with information, on a per-activity basis, about misconceptions to watch for along with specific steps to take to correct them. Teacher support materials for these activities are linked to higher-level support materials advising teachers in practical ways how to create an atmosphere of respect and inquiry in their classrooms.

Teachers' knowledge of mathematics content and how they apply that knowledge in the classroom impacts student learning.

Even in student-centered classrooms, the skills and knowledge of the teacher are of paramount importance. As John Van de Walle (2004) puts it,

What students learn is almost entirely dependent on the experiences that teachers provide every day in the classroom. To provide high-quality mathematics education, teachers must (1) understand deeply the mathematics they are teaching; (2) understand how children learn mathematics, including a keen awareness of the individual mathematical development of their own students; and (3) select instructional tasks and strategies that will enhance learning. (p. 3)

Researchers have known for a long time that the ability to do mathematics does not correlate well with the ability to teach it (Mewborn, 2000). Quantitative studies in the 1960s and 70s found no correlation between such factors as the number of mathematics classes a teacher had taken and student achievement. These studies were criticized for being crude, leading to subsequent studies that attempted to tease out the kinds of content knowledge that do matter.

And, indeed, the consensus now among researchers is that teachers don't need to learn more mathematics; they need greater conceptual understanding of the mathematics they already know (Mewborn, 2000). Denise Mewborn, in her meta-analysis of the available research (2000), notes that several available studies indicate that teachers who undergo training in conceptual ideas of mathematics are better equipped to engage students in the types of rich dialogue and experiences that lead students to deeper conceptual understandings (she cites, for example, Swafford, Jones, and Thorton, 1997, and Carpenter, Fennema, Peterson, and Carey, 1988). However, as Mewborn points out, even a deeper conceptual understanding of mathematics does not automatically translate to improved teaching. For that final step, teachers need to improve their understanding of how students learn mathematics.

Fortunately, according to the National Research Council (2001), professional development programs designed to transfer conceptual understanding to enhance teaching practices significantly increase student performance. The best evidence of the effectiveness of this approach, according to the NRC, is the work done on Cognitively Guided Instruction (CGI) at the University of Wisconsin-Madison. CGI places an emphasis on teaching teachers about the mathematical thinking of children. The NRC notes, "After teachers have studied the development of children's mathematical thinking, they tend to place a greater emphasis on problem solving, listen to their students more and know more about their students' abilities, and provide greater opportunity for their students to use a variety of solution methods. Gains in student achievement generally have been in the areas of understanding and problem solving, but none of the programs has led to a decline in computational skills, despite their greater emphasis on higher levels of thinking." (p.392).

How this principle is embodied in *Straight Curve Mathematics*

Straight Curve Mathematics supports teachers' conceptual knowledge of mathematics and how it applies to instruction in a couple of different ways.

Above all, *Straight Curve Mathematics* is supported by several levels of live, high-quality professional development.

Built into *Straight Curve Mathematics*, moreover, is a number of supports for teachers. As described above, embedded teacher support materials provide teachers with a variety of types of resources, including guidance about anticipated misconceptions and how to address them. Beyond the teacher support materials, every lesson begins, by default, with a Mini-lesson. The Mini-lesson is a direct instruction component, designed to guide teachers toward making successful presentations of the content. The Mini-lessons are flexible enough to support experienced teachers, but have enough embedded built-in support to guide teachers toward providing students with conceptually deep and correct instruction, all the while upgrading their own knowledge of mathematics concepts. Every Mini-lesson includes an on-call narrator that teachers can use to preview the instructional content to help them prepare for teaching it. The narrator, along with the teacher support materials, provides positive teaching models.

Mastery of mathematics concepts and principles requires extended, meaningful, and engaging practice.

The National Research Council (2001) has observed that one difference between reading and mathematics is that many—though certainly not all—students choose to spend significant amounts of free time reading. Whereas mathematics might occasionally come up in an incidental way during the lives of students, few choose to spend their free time doing extra mathematics. That means that the level of practice students receive in conjunction with their mathematics studies is critical.

Of course, one issue is that it simply takes time and practice to build proficiency. The NRC (2001) notes

Students need enough time to engage in activities around a specific mathematical topic if they are to become proficient with it. When they are provided with only one or two examples to illustrate why a procedure works or what a concept means and then move on to practice in carrying out the procedure or identifying the concept, they may easily fail to learn. To become proficient, they need to spend sustained periods of time doing mathematics—solving problems, reasoning, developing understanding, practicing skills—and building connections between their previous knowledge and new knowledge. (p. 135)

Part of achieving proficiency is building automaticity of low-level skills. In an age of calculators and computers, exactly what level of skills require automation is often debated (NRC, 2001), but nonetheless automating some mathematics skills allows students to concentrate their limited mental resources on higher-level problem solving.

Practice should be about more than low-level skills, though. Students should practice higher-level problem solving as well. The more students solve meaningful, interesting, open problems, the more they come to believe in themselves as budding mathematicians. The NRC (2001) contrasts this with students who are seldom challenged in this way: “In contrast, when students are seldom given challenging mathematical problems to solve, they come to expect that memorizing rather than sense making paves the road to learning mathematics, and they begin to lose confidence in themselves as learners” (p. 131). Happily, practice with low-level skills and practice with higher-level problem solving are not in conflict. Indeed, several studies show that even when teachers displace some of their students’ low-level practice with higher-level problem-solving practice, computational performance does not suffer (NRC, 2001).

So, students need both low-level and high-level practice. A great deal of research has gone into the phenomenon known as *flow*, which has been described by researcher Mihaly Csikszentmihalyi (1991) this way: “‘Flow’ is the way people describe their state of mind when consciousness is harmoniously ordered, and they want to pursue whatever they are doing for its own sake. In reviewing some of the activities that consistently produce flow—such as sports, games, art, and hobbies—it becomes easier to understand what makes people happy.” (p. 6). Flow

is the state of mind associated with immersive games and with engagement in immersive, interesting problems.

How this principle is embodied in *Straight Curve Mathematics*

In the spirit of providing both low- and high-level practice and in the spirit of providing forms of practice that promote flow, *Straight Curve Mathematics* features two significant kinds of practice. Investigations provide students with practice solving deep, open-ended problems, and Games provide students with practice gaining proficiency with lower-level skills.

Every Investigation starts with a short warm up that refreshes prior knowledge or teaches a new concept, then asks students to divide up into small groups to take on a challenge. These challenges are, by design, open-ended enough to allow multiple solution paths—some Investigations will even ask students to find more than one path to the solution. By regularly using *Straight Curve Mathematics*, students gain regular practice with higher-level problem solving. Investigations promote thoughtful solutions and deep thinking.

Games in *Straight Curve Mathematics*, on the other hand, promote the development of core skills. They are designed to be engaging and replayable, encouraging learners to return again and again to improve their scores or their times.

Assessment should provide teachers with meaningful information that guides their decisions (and students with meaningful information that guides their performance).

Assessment provides feedback for teachers and students alike. Without it, decisions are made in a vacuum.

It's surprising, then, that more isn't known about assessment practices in U.S. classrooms. According to the National Research Council (2001), "relative to the vast literature on external assessments and their results, little up-to-date information is available on how U.S. teachers conduct internal assessments in mathematics, particularly those activities such as classroom questioning, quizzes, projects, and informal observations" (p. 40). Still, based on available evidence, the NRC (2001) was able to recommend that teachers administer assessments "frequently and regularly in classrooms for the purpose of monitoring individual students' performance and adapting instruction to improve their performance" (p. 44).

Of course, collecting data and knowing what to do with it or how to interpret it are two different things. The NRC notes (2001), "Teachers' understanding of their students' work and the progress they are making relies on the teachers' own understanding of the mathematics and their ability to use that understanding to make sense of what the students are doing. Moreover, after interpreting students' work, teachers need to be able to use their interpretations productively in making specific instructional decisions: what questions to ask, tasks to pose, homework to assign. Studies show that when teachers learn to see and hear students' work during a lesson and use that information to shape their instruction, their instruction becomes clearer, more focused, and more effective" (p. 350). A more recent NRC report (2005) refers to formative assessments—frequent, ongoing assessments, as opposed to cumulative assessments—as being "essential" (p. 16).

One form of formative assessment is student self-assessment—students metacognitively monitoring their own learning. Brigid Barron, in conjunction with the Learning Technology Center at Vanderbilt (1998), advocates, based on long-term research with fifth graders, that problem-based learning activities ensure frequent opportunities for self-assessment: "An emphasis on self-assessment helps students develop the ability to monitor their own understanding and to find resources to deepen it when necessary" (p. 284).

The NRC points out (1999) that formative assessment can take nearly any form—informal questions, discussions, journaling, even well-designed multiple choice tests—but that a few critical factors need to be taken into account for it to be effective. Formative assessment must:

- Focus on conceptual understanding and not on facts or memorized procedures.

- Be engaged when teachers and students can learn from them and change course, rather than at the end just before moving on to the next topic.

Formative assessments that meet these criteria will result in greater learning and transfer (NRC, 1999).

How this principle is embodied in *Straight Curve Mathematics*

Every instructional block in each *Straight Curve Mathematics* lesson includes deliberate opportunities for assessment that allow both teachers and students to make informed decisions about their instructional course. These assessment opportunities are detailed in the table below.

Instructional Block	Assessment Opportunities
Mini-lesson	Every Mini-lesson is infused with frequent opportunities for practice through judged interactions. These interactions can take the form of fill-ins, drag-and-drops, multiple choice questions, or other interactions. When used by a teacher for large-group instruction, these interactions allow teachers to check in with students about their understanding. Teachers can ask individual students to offer answers to the questions and can ask students to articulate their reasoning. When used by individual students, the interactions give students valuable feedback about their progress. Further, every interaction is designed to give diagnostic feedback to students who choose incorrect answers so they can learn from their mistakes, capitalizing on a major teachable moment.
Investigation	Investigations normally ask teachers to divide the class up into small groups to solve the given problem. Along with each problem, students are given reflective prompts to respond to in their mathematics journal. These prompts are designed to help students reflect and assess their own learning process. Further, because students are solving problems in small groups, teachers are encouraged to circulate among the groups and listen critically to them problem solve. The teacher support materials provide teachers with guidance about misconceptions to listen for and how to address them.
Mathematics Workshop	The Math Workshop is where the groups come together and discuss their approaches to solving the problem posed in the Investigation. The teacher support materials provide teachers with guidance about questions to ask in order to assess the level of conceptual understanding of each group. The materials also provide guidance about common

	misconceptions to watch out for and how to address them.
Game	Not only are games more engrossing and fun than worksheets as a form of practice, they give continuous feedback to students about how they are performing, encouraging students to strive to improve.
Quiz	<p>Unlike paper quizzes, online Quizzes provide students with immediate performance feedback. Further, the first half of every Quiz is devoted specifically to practice for the test portion of the Quiz. The practice portion gives both targeted diagnostic feedback and on-call virtual mentor support.</p> <p>More importantly, Quizzes are designed to target a limited objective. Whereas most classroom tests cover multiple objectives, making it difficult to distinguish what students need the most help with, the focused Quizzes in <i>Straight Curve Mathematics</i> give teachers much more precise and useful information.</p>

In addition, every strand includes two kinds of pre-assessment options: readiness assessments and benchmark tests. Readiness assessments provide teachers information about whether students are prepared for the new material. Benchmark assessments provide teachers information about how much of the new material students already know. Benchmark tests are designed to go with the included cumulative tests to measure gains. See the embedded teacher support materials for more information about how to use these assessments.

Conclusion

Straight Curve Mathematics was developed to fill a specific need—to provide classroom teachers with resources that can help students develop a deeper conceptual understanding of mathematics. It was designed to specifically address three key areas identified as significant problems in elementary school mathematics instruction: shallow curricula, uninformative assessments, and under-prepared teachers.

Straight Curve Mathematics addresses these three problem areas through a research-based design that includes a deep curriculum that takes advantage of technology to provide interactive, engaging instruction and a strong focus on teacher-led instruction and embedded teacher support. This helps teachers:

- Go deeper with key mathematical concepts. Rather than try to address a broad array of objectives in a shallow way, *Straight Curve Mathematics* targets those objectives deemed hardest for students to learn and hardest for teachers to teach. The *Straight Curve Mathematics* instructional design team began with the Open Book analysis of mathematics strands (Educational Market Research, 2000) and from there engaged external subject matter experts to narrow objectives.
- Make informed instructional decisions. *Straight Curve Mathematics* includes several levels of formal assessment, as well as guidance to teachers about how to informally assess their students throughout the learning process.
- Become more comfortable with mathematics and with best practices for teaching mathematics. Embedded teacher support materials provide the right depth of information for teacher development. Further, *Straight Curve Mathematics* was designed to work in conjunction with expert, experienced professional development support provided by PLATO Professional Services.

Using the best available research, the design team for *Straight Curve Mathematics* was able to place emphasis on the teacher as instructional leader and utilize the best of technology to provide a resource that can dramatically improve academic performance in elementary mathematics classrooms.

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